The Thomson Jumping Ring Experiment and Ideal Transformer

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In this paper, we utilize the readily known theory of the ideal transformer to furnish a self-contained qualitative explanation on the a.c.-powered Thomson jumping ring (TJR) experiment.

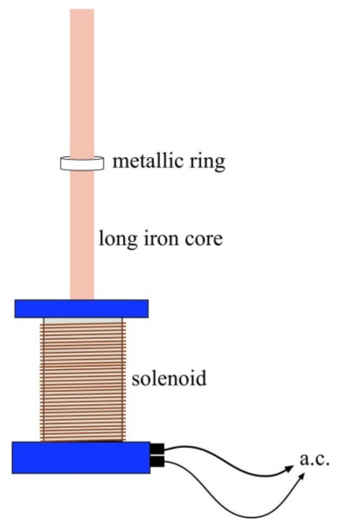

Fig. 1. A schematic diagram of the TJR experiment set-up.

The TJR experiment (Fig.1), a fascinating demonstration that always surprises audiences and arouses their curiosity, is usually performed in a classroom teaching of Faraday's law of electromagnetic induction or Lenz's law. As the a.c. power supply is turned on and the current in the lower thick solenoid is increased from zero, the upper metallic (usually aluminum) ring eventually floats up along the long iron core and stays at a height, which increases with the solenoid current and a.c. frequency. Most excitingly, the ring can be shot up to several meters high if the ring is pre-cooled by immersing it into liquid nitrogen.[1]

Lenz's law fails to explain

How come? We first try to attribute the striking repulsion to Lenz's law. A physics model of the set-up is the two coils shown in Fig. 2: coil I carries an a.c., and a current is induced in coil II. The spirit of Lenz's law is to oppose the change. When the magnetic flux through coil II is strengthening, the induced current in coil II will flow in a direction such that the magnetic force thus created will push coil II away from coil I, i.e., to a region of weaker field. But, if the magnetic flux is weakening, coil II tends to be attracted by coil I to a region of stronger field. Referring to the input sinusoidal curve shown in Fig. 2, the outcomes in the four quadrants are tabulated in Table 1.

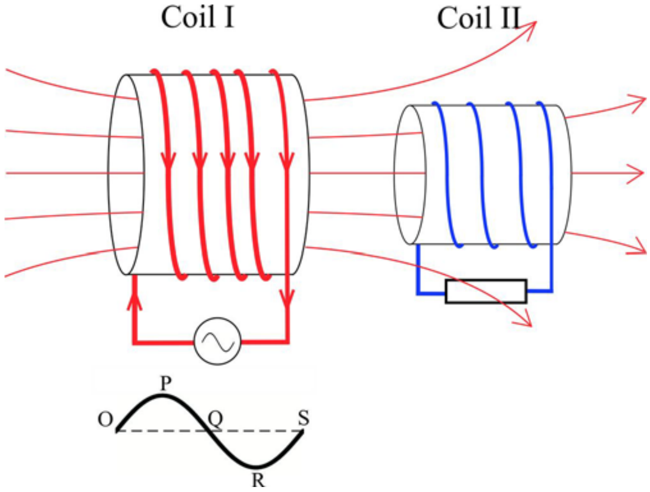

Fig. 2. A physics model of the TJR experiment set-up. The arrows shown correspond to an instantaneous direction of the a.c. and the B-field then produced.

Table 1. The four quadrants are referred to the sinusoidal curve shown in Fig. 2.

Quadrant	Flux through coil II	Force on coil II
OP	strengthening	repulsive
PQ	weakening	attractive
QR	strengthening	repulsive
RS	weakening	attractive

The force acting on coil II (the metallic ring) is repulsive, attractive, repulsive, and then attractive over the entire cycle OS. Therefore, if the a.c. frequency is f, the repulsion-attraction pair repeats itself at a frequency of $2f$. The ring has a mechanical inertia, so the movements of the ring caused by the individual repulsive and attractive forces will be largely negated if f is high enough. Therefore, when the solenoid is plugged into the mains (50-60 Hz), we only expect that the ring will, at most repeatedly jump and fall quickly with an exceedingly small amplitude, but never be repelled from the solenoid all the time! Hence, in this sense Lenz's law fails to explain the experiment; this result has been asserted in earlier studies.[2,3]

Ideal transformer

An ideal transformer, satisfying a few assumptions including no ohmic losses and no flux leakages, has an efficiency of 100%. The TJR experiment set-up is essentially a transformer: the thick solenoid is the primary coil, and the ring acts as a one-turn secondary coil; the internal resistance of the ring can be treated as a resistor loaded to the transformer. First, we need to review the relevant equations and concepts.

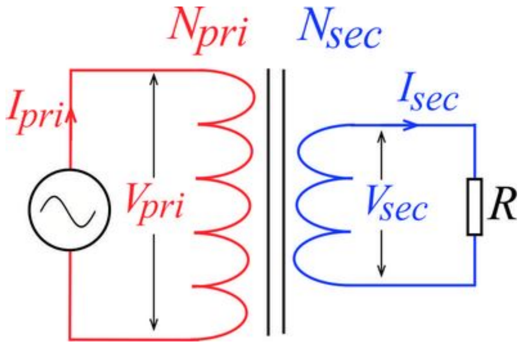

Fig. 3.   A schematic diagram of an ideal transformer.

Fig. 3 shows a schematic diagram of an ideal transformer. In introductory physics courses, students are taught that its main function is to step up or step down an alternating voltage, with the famous relationship

$$\frac{V_{pri}}{V_{sec}} = \frac{N_{pri}}{N_{sec}}, \tag{1}$$

where, as in the rest of this paper, the subscripts "*pri*" and "*sec*" stand for the primary and secondary coils, respectively; V is the voltage, and N is the number of turns. Because of Eq. (1) and the power relationship $V_{pri}I_{pri} = V_{sec}I_{sec}$,[4] we get the current ratio

$$\frac{I_{pri}}{I_{sec}} = \frac{N_{sec}}{N_{pri}}. \tag{2}$$

The secondary coil is loaded with a pure resistor R, therefore

$$V_{sec} = I_{sec}R. \tag{3}$$

(1) An argument leading to a crucial concept

If the load resistor R in Eq. (3) is reduced, the secondary current I_{sec} will be increased because V_{sec}, according to Eq. (1), only depends on the turns ratio and V_{pri}, which is in value equal to the source voltage. On the other hand, the current ratio (Eq. (2)) states that I_{pri} and I_{sec} are always in a simple ratio, meaning that when one of them is multiplied by a factor of, say, k, the other will be multiplied by this same factor too. A current will produce a magnetic field, so the magnetic flux through the iron core, denoted as Φ, is reasonably assumed to be produced by I_{pri} and/or I_{sec}, with a mathematical expression like $\Phi = \alpha I_{pri} + \beta I_{sec}$, where α and β are geometry-depending constants (since $B = \mu_o \times$ number of turns $\times$ current/length, and $\Phi = B \times$ area). Hence, when I_{pri} and I_{sec} are both multiplied by the factor k, Φ will become $k\Phi$. Next, a larger amplitude of the sinusoidal magnetic flux will produce a larger induced voltage across the secondary coil, V_{sec}. Simply put, if R is reduced, in sequence, I_{sec}, I_{pri}, Φ and V_{sec} will all be increased. But, $V_{sec} = I_{sec}R$, an increased V_{sec} will make I_{sec}, and then I_{pri}, Φ and V_{sec} all further increased. The loop runs infinitely, and finally, I_{sec}, I_{pri}, Φ and V_{sec} will all become infinite! This is nonsense, since V_{sec} is only a stepping up or stepping down of the source voltage.

In the argument, the only possible flaw is that the core flux is wrongly assumed to be produced by I_{pri} and/or I_{sec}, implying neither I_{pri} nor I_{sec} should produce the necessary core flux. Each of their fluxes is not zero, so we have to conclude that *their magnetic fluxes must always cancel each other out*, i.e., $\alpha I_{pri} + \beta I_{sec} = 0$. Actually, this flux cancellation is a well-known property of an ideal transformer,[6] but sadly, seldom discussed in the introductory texts.

(2) Magnetization Current

The remaining problem is, what actually causes the magnetic flux? In fact, the core flux is purely due to a third current, which, other than I_{pri}, circulates around the primary loop as well. This current, denoted as I_{mag} here, is customarily called the magnetization, exciting, or no-load (perhaps a misleading name) current. Some properties of this magnetization current are:[6] it is the same no matter the transformer is open-circuited, or loaded with a resistor of any value; it does not satisfy a current ratio like Eq. (2); it is $\pi/2$ lagging the source voltage; it delivers a zero time-averaged power from the source; it is generally small because of the large number of turns of the primary coil.

(3) The whole picture

According to the physics causality, the appearance of the three currents could be understood as follows. The a.c. source acts on the primary coil and produces the current I_{mag}, as same as the a.c. response of a pure inductor. This alternating I_{mag} produces a time-varying magnetic field to pass through the secondary coil to produce the induced voltage V_{sec}. As long as the output loop is a closed circuit, the secondary current I_{sec} appears, and then passes through the load resistor to produce an energy output. By conservation of energy, there must be a corresponding energy input. But, as in a pure inductor, the time-averaged power of I_{mag} is zero, so I_{mag} is ineligible to transport the energy. Eventually, the primary coil draws another current from the source to start the energy flow; this current is our familiar I_{pri}. In addition, the energy exchange between the input and output sides in the iron core requires the magnetic fluxes caused by I_{pri} and I_{sec} are always opposite and equal;[7] this is necessary to maintain proper functioning of the transformer, as our above argument shows.

In brief, I_{mag} is responsible for setting up the core flux, while (I_{pri}, I_{sec}) is responsible for the energy transport.

Transformer explanation

The two currents I_{pri} and I_{sec} suffice to give a simple qualitative explanation on the TJR experiment. The magnetic force unveiled by our preliminary analysis, as summarized in Table 1, is only the interaction between I_{mag} and I_{sec}. This force is periodically repulsive and attractive, resulting a null net force. The real cause is because of I_{pri} and I_{sec}, since the magnetic fields produced by them are *always* opposing each other.

As in the usual set-up, the metallic ring (secondary coil) is at the top and coaxial with the solenoid (primary coil), the cancellation of the fluxes by I_{pri} and I_{sec} is achieved when I_{pri} and I_{sec} always flow oppositely. It is known that two opposite currents repel each other. This is just the reason why the metallic ring is always repelled from the solenoid.

Two explanations

Indeed, the TJR experiment has already been well discussed and analyzed in the literature; the most accepted theory is to treat the ring as an *RL* circuit driven by the sinusoidally induced voltage.[2,3,8-12] Regarding the levitation of the ring, the *RL* method explains it is caused by the magnetic interaction of the whole solenoid current and the part of the ring current which exists because of the self-inductance of the ring, while the transformer method explains it is caused by the magnetic interaction of the whole ring current and I_{pri}, which is only a part of the solenoid current. These two explanations are each self-contained.

The most prominent advantage of the transformer explanation is that the self-inductance of the ring is not needed to be considered. The appearance of the self-inductance of a coil (L) in an equation implies the occurrence of a self-induction in that coil. In the transformer model, the flux produced by I_{sec} is always cancelled by that of I_{pri}, so I_{sec} is incapable of producing a usable flux to cause a self-induced current in the secondary coil. Hence, the self-inductance of the secondary coil (the ring) will not appear in the transformer explanation itself.

Non-ideal apparatus

Can the ideal transformer model truly explain the TJR experiment, of which the set-up is in fact not so ideal? This is really a critical question.

(1) Efficiency less than 100%

The efficiency of a real transformer is much less than 100%, but irrespective of this, a larger I_{sec} results a larger power output, prompting a larger power input and hence a larger I_{pri} (although I_{sec} and I_{pri} not satisfying the current ratio Eq. (2)). So, our argument leading to concluding the flux cancellation still holds.

(2) Solenoid's internal resistance

The internal resistance of the ring is treated as an output resistor *R*, so it is not neglected. The internal resistance of the thick solenoid (R_{sol}) exists as well – will the negligence of it cause a problem? The answer is no.

The levitation of the ring is caused by the interaction of the solenoid current (I_{sol}) and the ring current; the former is the root cause. The levitation is absolutely nothing to do with R_{sol}, as evidenced in the formulas.[3,10,11] So, with no surprise, TJR apparatuses equipped with, everything identical but solenoids of different R_{sol}'s, will produce the same levitation as long as the same I_{sol} flows in them. Among them, the physics of the ideal one ($R_{sol} = 0$) must be the simplest, but it is still a valid model provided the formula of the repulsive force derived from it is expressed in terms of I_{sol}.

(3)Flux leakage

The TJR apparatus has a conspicuous flux leakage. It is found experimentally that in going upward along the long iron core, the axial magnetic field leaks continuously to the outside and flares at the top of the core, forming a field pattern around the core,[3,8,11,12] which is nevertheless essential in producing the repulsive force. For example, if an upward force is produced, the magnetic field applied to the externally situated ring must have a radial outward component when I_{ring} flows clockwise, as viewed from above. The flux leaks, so at a higher position the flux through the ring will be smaller, thus causing a smaller ring current and a weaker repulsive force.[12] The ring will finally settle down at a height at where the repulsive force is balanced by the weight of the ring.

Inherently, the ideal transformer theory cannot consist with any flux leakages. In the next section, we will figure out, from first principles, an improved model in which the mutual inductance M is incorporated into the theory and can be parameterized to reflect any imperfect magnetic couplings between the two coils.

Derivations

Here, I_{pri}, I_{mag} and the product of them are derived. In the following, $\hat{I}$ is used to denote the amplitude of a current specified by its subscript. Suppose

$$I_{sec} = \hat{I}_{sec}\cos(\omega t). \tag{4}$$

We can express I_{pri} in terms of I_{sec} by using the current ratio $I_{pri} / I_{sec} = -N_{sec} / N_{pri}$.[4] We can find I_{mag} from the inductor equation (see subsection titled "*The whole picture*") $V_{pri} = -L_{pri} dI_{mag}/dt$, where V_{pri} can be expressed as $(N_{pri}/N_{sec})\ RI_{sec}$. It is straightforward to work out,

$$I_{pri} = -(\frac{N_{sec}}{N_{pri}})\hat{I}_{sec}\cos(\omega t), \tag{5}$$

and

$$I_{mag} = -\frac{R}{\omega L_{pri}}(\frac{N_{pri}}{N_{sec}})\hat{I}_{sec}\sin(\omega t). \tag{6}$$

Next, we go back and derive I_{pri} and I_{mag} again, but this time from first principles. Applying Kirchhoff's voltage law to the secondary coil, we have

$$RI_{sec} + L_{sec}\frac{dI_{sec}}{dt} + M\frac{dI_{tot\,pri}}{dt} = 0, \tag{7}$$

where $I_{tot\,pri}$ is the total primary current (don't confuse with I_{pri}), and M is the mutual inductance. Putting Eq. (4) into Eq. (7), we find $I_{tot\,pri} = I_{pri} + I_{mag}$, where

$$I_{pri} = -\frac{L_{sec}}{M}\hat{I}_{sec}\cos(\omega t), \tag{8}$$

and

$$I_{mag} = -\frac{R}{\omega M}\hat{I}_{sec}\sin(\omega t). \tag{9}$$

Using $N_{sec} / N_{pri} = \sqrt{L_{sec} / L_{pri}}$, and $M = \sqrt{L_{sec} L_{pri}}$ for a perfect magnetic coupling (no flux leakage), we can prove that Eq. (5) and Eq. (6) are identical to Eq. (8) and Eq. (9), respectively. Since I_{pri} and I_{mag} are orthogonal, $\hat{I}^2_{tot\,pri} = \hat{I}^2_{pri} + \hat{I}^2_{mag}$.

The time average of $I_{mag} \times I_{sec}$ is zero, while the magnitude of the time average of $I_{pri} \times I_{sec}$, expressed in terms of $\hat{I}_{tot\,pri}$, is

$$< I_{pri} \times I_{sec} >= \frac{\omega^2 M L_{sec}}{2(R^2 + \omega^2 L^2_{sec})} \hat{I}^2_{tot\,pri}, \qquad (10)$$

to which the repulsive force is proportional. As far as the TJR experiment is concerned, we change L_{sec} to L_{ring}, and $\hat{I}_{tot\,pri}$ to $\hat{I}_{sol}$. Eq. (10) agrees with the result derived by the *RL* method.[10,11,13] The mutual inductance M can be experimentally set to fit any real magnetic couplings between the two coils.

It is worthwhile to note that Eq. (8) can be rewritten as $M I_{pri} + L_{sec} I_{sec} = 0$, implying that the fluxes by I_{pri} and I_{sec} through the secondary coil are opposite and equal (cf. $\alpha I_{pri} + \beta I_{sec} = 0$ in our argument).

Conclusions

(1) The flux caused by our familiar I_{pri} cancels out that caused by I_{sec}. This should be generally true in practical resistor-loaded transformers.

(2) Even without any knowledge about the *RL* a.c. circuit (inductor's phase), students can still get a qualitative understanding on the TJR experiment if they know about the transformer's flux cancellation (opposition), which would be readily comprehensible with a simple argument like that introduced in this paper. In our opinion, perhaps this is the most valuable aspect of the (ideal) transformer explanation.

Acknowledgements

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4. The two voltages in Eq. (1) are *PD'*s, or both are *EMF'*s. The requirement "power input to the primary coil = power output from the secondary coil" corresponds to "$PD_{pri} \times I_{pri} = EMF_{sec} \times I_{sec}$". As $EMF = -PD$,[5] the most correct current ratio is negative. Ignoring the negative sign does not affect our qualitative reasoning.
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7. The input side transforms the input electrical energy to magnetic energy and store it in the core. At the same moment of its creation, all this magnetic energy is taken by the output side and transforms to the output electrical energy. The output takes away all this magnetic energy; no residue of the corresponding magnetic flux (energy) is left over.
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13. Assuming $I_{ring} = \hat{I}_{ring}\cos(\omega t)$ in Eq. (7), we will get $I_{tot\ pri} = \hat{I}_{tot\ pri}\cos(\omega t + \theta)$, where $\theta = \pi/2 + \tan^{-1}(\omega L_{ring} / R)$. Instead, we can assume $I_{tot\ pri} = \hat{I}_{tot\ pri}\cos(\omega t)$, and then find $I_{ring} = \hat{I}_{ring}\cos(\omega t - \theta)$. The former is the transformer (with *M*) method, while the latter is the standard *RL* method.