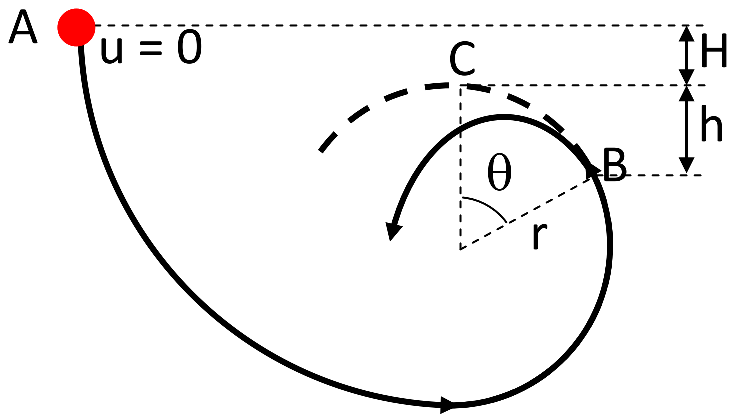
Condition for an Interrupted Pendulum Undergoing Complete Vertical Circular Motions



The bob is forbidden from going outside, but not inside. All frictional forces are neglected. The bob is released at A with the initial velocity u = 0.
By conservation of energy, the bob will attain the speed

$$v_B = \sqrt{2g(h+H)} \qquad \text{. . . . . (1)}$$

at B. Suppose the bob is at B, the string is still taut, so the net force acting on the bob towards the center is

$$T + mgcos\theta \qquad \text{. . . . . (2)}$$

where T is the tension in the string. If the bob can undergo a circular motion, it must satisfy the condition

$$net\ radial\ force = \frac{mv^2}{r}$$

Hence, by eq (1) and eq (2), the condition for continuing circular motion at B is

$T + mgcos\theta = \frac{mv_B^2}{r}$, or

$$T + mgcos\theta = \frac{2mg(h+H)}{r}. \qquad \text{.....(3)}$$

As the bob goes higher, h is smaller, but mgcosθ becomes larger. Therefore, the case

$$mgcos\theta > \frac{2mg(h+H)}{r}$$

may appear eventually if H is not large enough . If so, eq (3) will no longer be satisfied since the sting cannot be compressed, i.e., T $\geq$ 0. (cf., a rigid sick).

Hence, the bob is about to leave the vertical circle and go inside when

$$mgcos\theta = \frac{2mg(h+H)}{r} \qquad \text{.....(4)}$$

Since $h = r(1 - cos\theta)$ .....(5)

So, we get

$$cos\theta = \frac{2}{3}(1 + \frac{H}{r}) \qquad \text{.....(6)}$$

Eq (6) is used to calculate the angular position for the bob leaving the circle.

- In particular, if H = 0, $\theta = 48.2^0$

  If the bob does not leave throughout, $cos\theta \geq 1$, implying $H \geq \frac{r}{2}$.

- So, the least H for making complete circular motions is $H = \frac{r}{2}$.

  Then, at the top (h = 0, $\theta = 0^0$), T = 0 and the centripetal force required is purely provided by mg.